Algorithm #1

**Algorithm-1**

| Step | Cost of each execution | Total # of times executed |
| --- | --- | --- |
| 1 | 1 | 1 |
| 2 | 1 | (Q+P) |
| 3 | 1 | ∑L=PQ (Q – L + 2) |
| 4 | 1 | ∑L=PQ ∑U=LQ (Q – L + 1) |
| 5 | 1 | ∑L=PQ ∑U=LQ ∑1=LU (U – L + 2) |
| 6 | 6 | ∑L=PQ ∑U=LQ ∑1=LU (U – L + 1) |
| 7 | 3 | ∑L=PQ ∑U=LQ (Q – L + 1) |
| 8 | 2 | 1 |

Multiply col.1 with col.2, add across rows and simplify

T1(n) =

1+(Q+P) + (∑L=PQ (Q – L + 2)) + ( ∑L=PQ ∑U=LQ ∑1=LU (U – L + 2)) + 6(∑L=PQ ∑U=LQ ∑1=LU (U – L + 1)) + 3(∑L=PQ ∑U=LQ (Q – L + 1)) + 2

O(n3)

Algorithm #2

**Algorithm-2**

| Step | Cost of each execution | Total # of times executed |
| --- | --- | --- |
| 1 | 1 | 1 |
| 2 | 1 | (Q+P) |
| 3 | 1 | ∑L=PQ (Q – L + 1) |
| 4 | 1 | ∑L=PQ ∑L=PQ (U – L + 2) |
| 5 | 6 | ∑L=PQ ∑L=PQ (U – L + 1) |
| 6 | 3 | ∑L=PQ ∑L=PQ (U – L + 1) |
| 7 | 2 | 1 |

Multiply col.1 with col.2, add across rows and simplify

T2(n) =

1 + (Q+P) + (∑L=PQ (Q – L + 1)) + (∑L=PQ ∑L=PQ (U – L + 2)) + 6(∑L=PQ ∑L=PQ (U – L + 1)) + 3(∑L=PQ ∑L=PQ (U – L + 1)) + 2

= O(n2)

Algorithm #3

**Algorithm-3**

| Step | Cost of each execution | Total # of times executed in any single recursive call |
| --- | --- | --- |
| 1 | 3 | 1 |
| 2 | 3 | 1 |
| Steps executed when the input is a base case: 1- 2 | | |
| First recurrence relation: T(n=1 or n=0) = 12 | | |
| 3 | 5 | 1 |
| 4 | 2 | 1 |
| 5 | 1 | (n/2)+1 |
| 6 | 6 | n/2 |
| 7 | 4 | n/2 |
| 8 | 2 | 1 |
| 9 | 1 | (n/2)+1 |
| 10 | 6 | n/2 |
| 11 | 4 | n/2 |
| 12 | 4 | 1 |
| 13 | 5 | 1 |
| 14 | 6 | 1 |
| 15 | 5 | 1 |
| Steps executed when input is NOT a base case: 1, 3-15 | | |
| Second recurrence relation: T(n>1) = 2T(n/2) + 12n + 37 | | |
| Simplified second recurrence relation (ignore the constant term): T(n>1) = 2T(n/2) + 12n | | |

Solve the two recurrence relations using any method (recommended method is the Recursion Tree). Show your work below:

T(n) = 1+ 1 + (n+1) + 8n + 4n + 2 = (n+1)+(12n+4) = 13n + 5 = O(n)

Algorithm #4

**Algorithm-4**

| Step | Cost of each execution | Total # of times executed |
| --- | --- | --- |
| 1 | 1 | 1 |
| 2 | 1 | 1 |
| 3 | 1 | (Q+P) |
| 4 | 6 | ∑I=PQ (Q – L + 1) |
| 5 | 3 | ∑I=PQ (Q – L + 1) |
| 6 | 2 | 1 |

Multiply col.1 with col.2, add across rows and simplify

T4(n) =

1 + 1+ (Q+P) + 6(∑I=PQ (Q – L + 1)) + 3(∑I=PQ (Q – L + 1)) + 2 = O(n)